CHIZIQLI OPERATORLARNING TURLI BAZISLARDAGI MATRITSALARI ORASIDAGI BOG'LANISH.

Turdiboyev Sanjar Sobirjon o'g'li

JDPU, Matematika va Informatika fakulteti, katta o'qituvchisi

Jalolova Muxlisa Sadriddin qizi,

Jumaboyeva Sarvinoz Ilhom qizi,

JDPU, Matematika va Informatika fakulteti 3-bosqich talabasi

Annotatsiya. Chiziqli operatorlar chiziqli algebra asosiy tushunchalaridan biri bo'lib, ular fazolarni va ularning elementlarini o'zaro bog'laydigan matematik ob'ektlardir. Chiziqli operatorlar va ularning matritsa ko'rinish, O'zaro teskari chiziqli operatorlar, Chiziqli operatorlar va bazis o'zgarishi. Chiziqli operator tushunchasi va ularning asosiy xossalari.

Kalit so'z. Chiziqli operator, algebra, bazis, teskari matrisa, bazisdagi matrisalar.

Chiziqli V fazoda berilgan bazisdagi chiziqli operatorlarni matritsalari. V fazodagi $\overline{e_1}, \overline{e_2}, ...\overline{e_n}$ bazisni fiksirlaymiz, x V dagi ixtiyoriy element va $x = \sum_{k=1}^{n} x^k e_k$

Esa bu x elementi berilganbazisdagi yoyilmasi hamda A esa L(V,V) dagi chiziqli operator bo'lsin u holda (1) dan $Ax = \sum_{k=1}^{n} x^k Ae_k$ $\quad Ae_k = \sum_{j=1}^{n} a_k^j e_j$

Deb olsak, quyidagicha yozamiz: $Ax = \sum_{k=1}^{n} x^k \sum_{j=1}^{n} a_k^j e_j = \sum_{j=1}^{n} \left(\sum_{k=1}^{n} a_k^j x^j \right) e_j$

Shunday qilib, $y = Ax$ va $y = \left(y^1, y^2, ...y^n\right)$ elementning koordinatalari bo'lsa u holda $$y^j = \sum_{k=1}^{n} a_k^j x^j, \; j = 1, 2, ..., n$$

Ushbu $A = \left(a_k^j\right)$ kvadrat matrisani qaraylik, bu u matritsa berilgan $\overline{e_1}, \overline{e_2}, ...\overline{e_n}$ bazisdagi A chziqli operatorning matritsasi deyiladi. Oldingi ko`rsatilgan usul bilan birgalikda uni berilgan bazisdagi matritsaviy yozuvi ham ishlatiladi: $y = Ax$

Agar $x = \left(x^1, x^2, ...x^n\right)$ bo`lsa, u holda $y = \left(y^1, y^2, ...y^n\right)$ dagi $y^j, \; j = 1, 2, ..., n$ formula orqali A ning a_k^j elementlari esa formula orqali hisoblanadi.

Agar A operator nol operator bo`lsa, u holda bu operatorning A matritsasining barcha elementlari ixtiyoriy bazisda nollardan iborat, ya`ni A matritsa nol matritsa bo`ladi.

Agar A operator birlik operator bo`lsa, ya`ni A= I bo`lsa, u holda bu operatorning ixtiyoriy bazisdagi matritsasi birlik matritsadan iborat bo`ladi, ya`ni A= E .

1-teorema. V chiziqli fazoda $\overline{e_1}, \overline{e_2}, ...\overline{e_n}$ bazis berilgan va $A = \left(a_k^j\right)$ n- tartbli kvadrat matritsa bo`lsin, u holda A shunday yagona chiziqli operator mavjudki, bu A matritsa berilgan bazisda ushbu operatorni matritsasi bo`ladi.

A va B matritsalar n tartibli kvadrat matritsalar bo`lsin. A va B V fazoda ularga mos $\{e_k\}$ bazisdagi operatorlar bo`lsin, u holda teoremaga ko`ra $A+\lambda B$ matritsaga $A+\lambda B$ operator mos keladi. Bunda λ -biror son.

2-teorema. A chiziqli operatorning $rangA$ rangi matritsasi rangiga teng.

1-natija. A va B matritsalar ko'paytmasining rangi quyidagi munosabatlarni bajaradi: $rangAB \le rangA, rangAB \le rangB, rangAB \ge rangA + rangB - n.$

2-natija. A operator uchun teskari A^{-1} operator faqat va faqat A operator matritsasining rangi n ga (n= dimV) teng bo'lgandagina mavjud bo'ladi. Bu holda A matritsaga teskari A^{-1} matritsa ham mavjud bo'ladi. Endi yangi bazisga o'tganda chiziqli operator matritsasini almashtirishni qaraylik. V chiziqli fazo, A esaL(V,V) dagi chiziqli operator $\overline{e_1}, \overline{e_2}, ...\overline{e_n}$ va $e_1, e_2, ...e_n$ V dagi 2 ta bazis hamda

$$e_k = \sum u_k^i e_i , \quad k = 1, 2, ..., n$$

Esa $\{e_k\}$ bazisdan $\{\tilde{e}_k\}$ bazisga o'tish formulasi bo'lsin $U = \left(u_k^i\right)$ deb olamiz, $rangU = n$ ga teng $A = \left(a_k^j\right)$ va $A = \left(a_k^j\right)$ matritsalar A operatorni $\{e_k\}$ va $\{\tilde{e}_k\}$ bazislardagi matritsalari bo`lsin Bu matritsalar orasidagi munosabatni topamiz.

3-teorema. A operatorni $\{e_k\}$ va $\{\tilde{e}_k\}$ bazislardagi $A = \left(a_k^j\right)$ va $A = \left(a_k^j\right)$ matritsalari orasida $A = U^{-1} AU$ munosabat mavjud.

$A = U^{-1}AU$ formulani ikkala tomonini o`ngdan $U^{-1}$ va chapdan U ga ko`paytirib, quyidagi tenglikni hosil qilamiz: $A = UAU^{-1}$

A va B n- tartibli kvadrat matritsalar. A va B lar $\{e_i\}$ bazisdagi ularni mos operatorlari bo`lsin. U holda $A + \lambda B$ matritsaga $A + \lambda B$ chiziqli operator mos keladi. Yuqoridagi teoremadan $\det A = \det A$ kelib chiqadi. Shunday qilib, chiziqli operatorning matritsasini determinanti bazisni tanlab olishga bog`liq emas. Shu sababli A chiziqli operatorning determinanti det A tushunchasini kiritish mumkin, $\det A = |A|$ A- operatorning ixtiyoriy bazisdagi matritsasi.

Fazoning ikkita $\overline{e_1}, \overline{e_2}, ...\overline{e_n}$ $\overline{f_1}, \overline{f_2}, ..., \overline{f_n}$

Bazisi va bitta φ chiziqli operatorini olamiz. Bu φ operatorining a va b bazislardagi matrisalari

$$A = \begin{pmatrix} a_{11} & a_{12}\dots & a_{1n} \\ a_{21} & a_{22}\dots & a_{2n} \\ a_{n1} & a_{n2}\dots & a_{nn} \end{pmatrix} \text{va}\, B = \begin{pmatrix} b_{11} & b_{12}\dots & b_{1n} \\ b_{21} & b_{22}\dots & b_{2n} \\ b_{n1} & b_{n2}\dots & b_{nn} \end{pmatrix} \text{bo'lsin.}$$

Bu matrisalarni aniqlovchi tengliklar qisqacha bunday yoziladi:

$$\begin{cases} \varphi\overline{e_k} = \sum_{i=1}^{n} a_{ik}\overline{e_i}\left(k = \overline{1,n}\right) \\ \varphi\overline{f_k} = \sum_{i=1}^{n} b_{ik}\overline{f_i}\left(k = \overline{1,n}\right) \end{cases}$$ bazisni bazis orqali chiziqli ifodalaymiz:

$$\begin{cases} \overline{f_1} = c_{11}\overline{e_1} + c_{21}\overline{e_2} + ... + c_{n1}\overline{e_n}, \\ \overline{f_2} = c_{12}\overline{e_1} + c_{22}\overline{e_2} + ... + c_{n2}\overline{e_n}, \\ .... \\ \overline{f_n} = c_{1n}\overline{e_1} + c_{2n}\overline{e_2} + ... + c_{nn}\overline{e_n}, \end{cases}$$ sistemaning $C = \begin{pmatrix} c_{11} & c_{12}\dots & c_{1n} \\ c_{21} & c_{22}\dots & c_{2n} \\ c_{n1} & c_{n2}\dots & c_{nn} \end{pmatrix}$ matrisasi

xosmasdir.” Agar $\overline{e_1}, \overline{e_2}, ...\overline{e_n}$ vektorlar sistemasi fazoning bazisi va $\overline{f_1}, \overline{f_2}, ..., \overline{f_n}$ lar shu fazoning ixtiyoriy vektorlari bo’lsa, unda shunday yagona φ operator mavjudki, u $\overline{e_1}, \overline{e_2}, ...\overline{e_n}$ bazis sistemasini $\overline{f_1}, \overline{f_2}, ..., \overline{f_n}$ larga o’tkazadi” degan teoremaga binoan yagona ν chiziqli operator mavjud bo’lib, u (1) bazis vektorlarni (4) vektorlarga akslantiradi: $\nu\overline{e_i} = \overline{f_i}\,\left(i = \overline{1,n}\right)$ ning ikkala tomoniga φ operatorni tatbiq etamiz. Natijada hosil bo’ladi.

Oxirgi tenglamalarning o'ng tomonidagi $\varphi \overline{f_i}$ $\left(i=\overline{1,n}\right)$ larni bilan almashtirsak, $\varphi v \overline{e_i} = \sum_{i=1}^{n} b_{ik} \overline{f_i}$ kelib chiqadi. Agar $\overline{f_i}$ $\left(i=\overline{1,n}\right)$ larning o'rniga qo'ysak, natijada quyidagiga ega bo'lamiz: $\varphi v \overline{e_k} = \sum_{i=1}^{n} b_{ik} v \overline{e_i}$. v ning$|C|$ detriminantni 0 dan farqli bo'lgani sababli, v ga teskari v^{-1} operator mavjud bo'lib, uni (6) vektorga tatbiq etamiz:

$$v^{-1}\varphi \overline{e_k} = v^{-1}\sum_{i=1}^{n} b_{ik} v \overline{e_i} = \sum_{i=1}^{n} v^{-1} b_{ik} v \overline{e_i} = \sum_{i=1}^{n} b_{ik} v^{-1} v \overline{e_i} = \sum_{i=1}^{n} b_{ik} \varepsilon \overline{e_i} = \sum_{i=1}^{n} b_{ik} \overline{e_i}$$

$v^{-1}\varphi \overline{e_k} = \sum_{i=1}^{n} b_{ik} \overline{e_i}$ (ε-birlik operator).

Bir tomondan $v^{-1}\varphi v$ operatorning bazisdagi matrisasi $C^{-1}AC$ bo'lib (chunki $v^{-1} \to C^{-1}$, $\varphi \to A$ va $v \to C$) ikkinchi tomondan, bu operatorning bazisdagi matrisasi B bo'lganligi sababli $B = C^{-1}AC$ bo'ladi.

Misol. Uch o'lchovli arifmetik V fazoning

$$\overline{e_1} = (1,0,0), \overline{e_2} = (0,1,0), \overline{e_3} = (0,0,1),$$
$$\overline{f_1} = (1,1,1), \overline{f_2} = (1,2,1), \overline{f_3} = (2,-1,1),$$

Bazislarni va φ $(a_1,a_2,a_3) = (a_1,2a_2,3a_3)$ operatorni olamiz. Bu operatorning birinchi bazisdagi matrisasi $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ bo'lib, ikkinchi bazisning birinchi basis orqali chiziqli ifodasi quyidagidan iborat:

$$\overline{f_1} = \overline{e_1} + \overline{e_2} + \overline{e_3},$$
$$\overline{f_2} = \overline{e_1} + 2\overline{e_2} + \overline{e_3},$$
$$\overline{f_3} = 2\overline{e_1} - \overline{e_2} + \overline{e_3},$$

Demak, $C = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & -1 \\ 1 & 1 & 1 \end{pmatrix}$ va $C^{-1} = \begin{pmatrix} -3 & -1 & 5 \\ 2 & 1 & -3 \\ 1 & 0 & -1 \end{pmatrix}$ lardan iborat bo'lgan uchun φ operatorning ikkinchi bazisdagi matrisasi $B = C^{-1}AC = \begin{pmatrix} 10 & 8 & 11 \\ -5 & -3 & -7 \\ -2 & -2 & -1 \end{pmatrix}$ bo'ladi.

FOYDALANILGAN ADABIYOTLAR.

1. R.N.Nazarov, B.T.Toshpo'latov, A.D.Do'sumbetov. Algebra va sonlar nazariyasi. I qism. Toshkent. O'qituvchi. 1993 y.
2. B.Abdalimov, A.Abdug'apporov, M.Musamuhammedov, S.Toshpo'latov. Oliy matematikadan masalalar yechish bo'yicha qo'llanma. "O'qituvchi"., Toshkent 1985 y.
3. M.A. Mirzaaxmedov, Sh.N.Ismailov, A.Q.Amanov. Algebra va analiz asoslari geometriya II- qism. O'rta ta'lim muassasalarining 11-sinf va o'rta maxsus, kasb-hunar ta'limi muassasalari o'quvchilari uchun darslik. 1-nashr. "O'zbekiston". Toshkent 2018y.
4. Turdiboyev S. DEVELOPING MATHEMATICAL AND LOGICAL KNOWLEDGE OF SCHOOL PUPILS //Архив Научных Публикаций JSPI. – 2020.
5. Turdiboyev S. ПРИМЕНЕНИЕ ДИФФЕРЕНЦИАЛЪНЫХ МОДЕЛЕЙ ПРИ ИЗУЧЕНИИ ЕСТЕСТВЕННЫХ НАУК //Архив Научных Публикаций JSPI. – 2020.
6. Turdiboyev S., Jasur B. FUNKSIYA HOSILASINING TADBIGI //Журнал математики и информатики. – 2022. – Т. 2. – №. 1.
7.Majidov, Jasur Baxtiyarovich (2022). SOTSIAL INTELLEKT VA UNING INSON HAYOTIDAGI O'RNI. Oriental renaissance: Innovative, educational, natural and social sciences, 2 (2), 608-613.